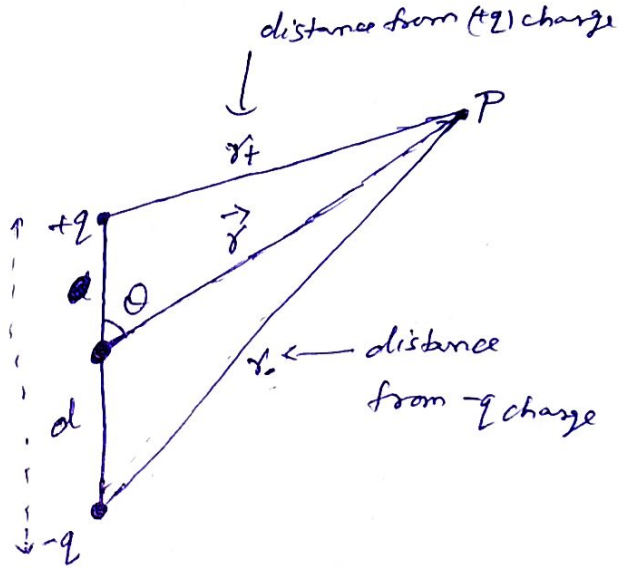
B.Sc. Physics Part-II
Paper-IV, Group-A

Approximate potentials at large distances:

- Consider an example of electric dipole which consists of two equal and opposite charges ($\pm q$) separated by a distance d. Find the approximate potential at points far from the dipole.

Solution.



Potential

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{r_+} - \frac{q}{r_-}\right]$$

Next, we use the law of cosines to write

$$r_\pm^2 = r^2 + \left(\frac{d}{2}\right)^2 \mp rd\cos\theta$$

~~$= r^2 [\ldots]$~~

or $$r_\pm^2 = r^2\left[1 \mp \frac{d}{r}\cos\theta + \frac{d^2}{4r^2}\right]$$

Since we are intrested in the regime $r \gg d$.

Therefore $\frac{d^2}{4r^2}$ is negligible

$$r_\pm^2 \simeq r^2\left[1 \mp \frac{d}{r}\cos\theta\right]$$

or $$\frac{1}{r_\pm} \simeq \frac{1}{r}\left[1 \mp \frac{d}{r}\cos\theta\right]^{-\frac{1}{2}}$$

Using binomial expansion in above expression, we obtain

$$\frac{1}{r_\pm} \simeq \frac{1}{r}\left[1 \pm \frac{d}{2r}\cos\theta\right]$$

Now we can write

$$\frac{1}{r_+} - \frac{1}{r_-} \simeq \frac{1}{r}\left[1 + \frac{d}{2r}\cos\theta - 1 + \frac{d}{2r}\cos\theta\right]$$

$$\simeq \frac{1}{r}\left[\frac{d}{r}\cos\theta\right]$$

$$\simeq \frac{d}{r^2}\cos\theta$$

The potential $V(\vec{r})$, therefore, is given by

$$\boxed{V(\vec{r}) \simeq \frac{1}{4\pi\epsilon_0}\frac{q d\cos\theta}{r^2}}$$

This example can be extended to obtain approximate potential of an arbitrary localized charge distribution. We will obtain this relation in next class note.